Complicated Colorings

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The Sierpiński partition

The study of strong colorings was born with the 1933 Sierpiński partition. He proved that Ramsey's theorem fails at the level of ω_1. Specifically, there exists a coloring $c : [\omega_1]^2 \to 2$ such that for every uncountable $A \subseteq \omega_1$, we have $c``[A]^2 = 2$.

The Sierpiński partition, cont.

Write $S := \{\alpha + 1 \mid \alpha < \omega_1\}$ and $L := \omega_1 \setminus S$.
Let $\{r_\alpha \mid \alpha \in S\}$ injectively enumerate some subset of $(-\infty, 0)$.
Let $\{r_\alpha \mid \alpha \in L\}$ injectively enumerate some subset of $(0, \infty)$.

For $\alpha < \beta < \omega_1$, set $c(\alpha, \beta) := 1$ iff $r_\alpha < r_\beta$.

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For $\alpha < \beta < \omega_1$, set $c(\alpha, \beta) := 1$ iff $r_\alpha < r_\beta$.
Since $\mathbb{R}$ is separable, for every $A \in [\omega_1]^{\omega_1}$, we have $c``[A]^2 = 2$.

A finer concept

Definition (Shelah)

$\mathrm{Pr}_1(\lambda, \theta, \chi)$: $\exists d : [\lambda]^2 \to \theta$ such that for every $\mathcal{A} \subseteq [\lambda]^{<\chi}$ of size λ, consisting of pairwise disjoint sets, and every color $\gamma < \theta$, there exist a, b in $\mathcal{A}$ with $\sup(a) < \min(b)$ satisfying $c[a \times b] = \{\gamma\}$.

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Is c a witness to $\mathrm{Pr}_1(\omega_1, 2, 3)$?

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So, the Sierpiński partition c is a witness to $\mathrm{Pr}_1(\omega_1, 2, 2)$.

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Is c a witness to $\mathrm{Pr}_1(\omega_1, 2, 3)$?

No. Take $\mathcal{A} := \{\{\alpha, \alpha+1\} \mid \alpha \in L\}$. Now, if $\alpha < \beta$ in L, then $c([\{\alpha, \alpha+1\} \times \{\beta, \beta+1\}] = \{c(\alpha, \beta+1), c(\alpha+1, \beta)\} = 2$.

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We have seen that the parameter χ makes a difference. What about the parameter θ?

A finer concept, the θ parameter

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In the 1990's, Shelah proved that $\mathrm{Pr}_1(\lambda^+, \mathrm{cf}(\lambda), \mathrm{cf}(\lambda))$ holds for every singular cardinal λ.

Whether $\mathrm{Pr}_1(\lambda^+, \lambda, 2)$ holds for every singular cardinal λ is the oldest open problem of this field.

$\lambda = \aleph_1$

Theorem (Sierpiński, 1933)

$\mathrm{Pr}_1(\aleph_1, 2, 2)$.

Theorem (Galvin-Shelah, 1973)

$\mathrm{Pr}_1(\aleph_1, 4, 2)$.

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Fact (Galvin)
$\mathrm{MA}_{\aleph_1}$ refutes $\mathrm{Pr}_1(\aleph_1, \aleph_1, \omega)$.

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Theorem (Todorcevic, 1987/ Shelah [Sh:276])

If $\lambda > \aleph_1$ regular, and admits a nonreflecting stationary set, then $\mathrm{Pr}_1(\lambda, \lambda, 2)$.

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If $\lambda > \aleph_1$ regular, and admits a nonreflecting stationary set, then $\mathrm{Pr}_1(\lambda, \lambda, 2)$.

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If $\lambda > \aleph_2$ regular, and $E^{\lambda}_{\geq \aleph_2}$ admits a nonreflecting stationary set, then $\mathrm{Pr}_1(\lambda, \lambda, \omega)$.

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"Remark. Is this hard? A posteriori it does not look so, but we have worked hard on it several times without success (worse: produced several false proofs)."

Shelah's opening of [§1, Sh:572].

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Main Result

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Definition

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there exist $\alpha < \beta < \kappa$ with $\varphi(\alpha) = \varphi(\beta)$ such that $c(\eta {}^\frown \rho) = \ell(\eta)$ for all $\eta \in u_\alpha$ and $\rho \in v_\beta$.

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The proof takes advantage of oscillation features of a simple weight function for C-sequences. Plus, at some point, it splits into two:

- if $\chi^{<\chi} > \chi$, we use a particular club guessing at $E^{\chi^+}_{\chi}$;
- if $\chi^{<\chi} = \chi$, we use an higher analogue of Todorcevic's square-bracket operation from the complete binary tree ${}^{<\omega}2$.

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If $\lambda > \kappa \geq \chi$ regular cardinals, $\mathrm{P}\ell_6(\kappa, \chi)$ holds, and $E^\lambda_{\geq\chi}$ admits a nonreflecting stationary set, then $\mathrm{Pr}_1(\lambda, \lambda, \chi)$.

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Proof.

We fix a witness $c : {}^{<\omega}\kappa \rightarrow \omega$ to $\mathsf{P}\ell_6(\kappa, \chi)$, and shuffling surjections $g : \lambda \rightarrow \lambda, h : \lambda \rightarrow \kappa$.

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Recalling that $\mathrm{P}\ell_6(\chi^+, \chi)$ holds for every regular χ, we get:

Main Result

If λ, χ regular cardinals, $\lambda > \chi^+$, and $E^{\lambda}_{\geq\chi}$ admits a nonreflecting stationary set, then $\mathrm{Pr}_1(\lambda, \lambda, \chi)$.