Logic Colloquium 2009
Sofia, Bulgaria, July 31 – August 5

▶ ASSAF RINOT, *Diamond, non-saturation, and weak square principles.*
School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israel.
URL: `http://www.tau.ac.il/~rinot`

We report on results from [1] and [2] concerning the effect of weak square principles to guessing principles. Let Refl_λ denote the assertion that every stationary subset of $\{\alpha < \lambda^+ \mid \mathrm{cf}(\alpha) = \mathrm{cf}(\lambda)\}$ reflects. A corollary to the results that we shall discuss in our talk is the following.

Theorem. *For a singular cardinal λ:*

(1) $\mathrm{GCH} + \mathrm{Refl}_\lambda + \square^*_\lambda \Rightarrow \diamondsuit^*_{\lambda^+}$;
(2) $\mathrm{GCH} + \mathrm{Refl}_\lambda + \mathrm{SAP}_\lambda \not\Rightarrow \diamondsuit^*_{\lambda^+}$;
(3) $\mathrm{GCH} + \mathrm{Refl}_\lambda + \mathrm{SAP}_\lambda \Rightarrow \diamondsuit_S$ *for every stationary* $S \subseteq \lambda^+$;
(4) $\mathrm{GCH} + \mathrm{Refl}_\lambda + \mathrm{AP}_\lambda \not\Rightarrow \diamondsuit_S$ *for every stationary* $S \subseteq \lambda^+$.

In addition, we prove that SAP_λ (and hence $\square^*_\lambda$) implies that $\mathrm{NS}_{\lambda^+} \upharpoonright S$ is non-saturated for every $S \subseteq \lambda^+$ that reflects stationarily often. We prove that the failure of a guessing principle introduced by Džamonja and Shelah is equivalent to the failure of Shelah's strong hypothesis. We also provide two (negative) answers to a question of König, Larson and Yoshinobu; one in the presence of GCH, and the other in its absence.

References

[1] M. Gitik and A. Rinot. The failure of diamond on a reflecting stationary set. *preprint,* 2009.
[2] A. Rinot. A relative of the approachability ideal, diamond and non-saturation. *preprint,* 2009.